

# No Supersymmetry without Supergravity

## Induced Supersymmetry Representations on Composite Effective Superfields

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### Abstract

Induced supersymmetry representations on composite operators are studied. In superspace the ensuing transformation rules (trivially) lead to an effective superfield. On the other hand, an induced representation must exist for non-linear (“on-shell”) supersymmetry as well. As this choice of the representation is physically irrelevant, any formulation of an effective action starting from the superspace representation must equally well be possible in a non-linear representation. We show that this leads to very relevant constraints on the formulation of effective actions in terms of composite operators. These ideas are applied to the simplest case of such a theory,  $N = 1$  SYM. It is shown that soft supersymmetry breaking within that theory forces one to include besides the Lagrangian multiplet  $S$  all currents of the super-conformal structure, embedded in a supergravity background, as relevant fields.

*PACS:* 11.15.Kc; 11.30.Pb; 11.30.Qc

*keywords:* SYM theory; non-linear representations; low-energy effective action

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# 1 Introduction

$N = 1$  SYM theories attracted much attention during the last year. On the one hand this is due to substantial progress in the understanding of supersymmetric gauge theories including  $N = 1$  SYM as a sub-sector, based on the study of string theory/matrix models as well as field-theoretic methods [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]. On the other hand first results from lattice simulations are available [11, 12, 13], which should allow tests of analytic results in the near future. However, there could arise serious difficulties in the comparison of the two approaches to SYM theories: Mostly analytic calculations are restricted to the study of the superpotential, as its holomorphic dependence is one of the most important constraints within supersymmetric theories. Also the set of physical fields considered therein [14, 15] might not be general enough: In most applications there appear only the gluino bilinear operator with its fermionic partner generating the low energy spectrum.

In two recent papers [16, 17] the authors have shown that the restriction to determine the superpotential and the difficulty of the missing glue-balls are actually related. Two operators generating glue-balls,  $\text{Tr } F_{\mu\nu} F^{\mu\nu}$  and  $\text{Tr } F_{\mu\nu} \tilde{F}^{\mu\nu}$ , are part of the effective superfield  $S \propto \text{Tr } W^\alpha W_\alpha$ , but they appear as *highest* component, i.e. at a place where one usually expects an auxiliary field. As long as the explicit calculations are restricted to the superpotential  $W(S)$ , this “auxiliary” field is not integrated out explicitly, as this step would require detailed knowledge about the non-holomorphic part of the effective Lagrangian. Nevertheless, it is assumed *implicitly* in most cases, that the highest component of  $S$  is auxiliary: If the first derivatives  $W_{,i}$  of the superpotential shall define the minimum and the second derivative  $W_{,ij}$  the spectrum, then all highest components of the involved chiral fields *must be standard auxiliary fields*. Thus the glue-ball pair-operators drop out of the action<sup>1</sup>.

We briefly want to recall why this step cannot be consistent from fundamental physical considerations. In any supersymmetric theory formulated in superspace there appear auxiliary fields. E.g. in the Wess-Zumino model the highest component of the chiral field  $\Phi_{WZ} = \varphi + \theta\psi + \theta^2 F_{WZ}$  obeys

$$\langle \Omega | T F_{WZ}(x) F_{WZ}(y) | \Omega \rangle \propto \delta(x - y) . \quad (1.1)$$

This relation holds in the full quantum theory as a consequence of supersymmetry Ward-identities. Thus the action of the Wess-Zumino model is ultra-local in the classical as well as in the quantum theory and, as a consequence,  $F_{WZ}$  may be eliminated without changing the physics of the model<sup>2</sup>.

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<sup>1</sup>Contrary to this assumption  $S$  is often called “glue-ball superfield”.

<sup>2</sup>Here we consider the renormalizable linear model, only. In that case the integration over the auxiliary field in the path integral does not lead to a non-trivial functional determinant. Of course, this is not true in a non-linear model and thus algebraic equations of motion are not sufficient to ensure at the quantum level the equivalence of a theory with and without auxiliary fields, resp. However, the present discussions about the interpretation of auxiliary fields does not concern this complication.

In its confined region,  $N = 1$  SYM theory is described by a chiral field  $S \propto \text{Tr } W^\alpha W_\alpha = \varphi + \theta\psi + \theta^2 F_{SYM}$  as well, the Lagrangian multiplet. This is not a fundamental field, but a superfield of composite operators. Thus the two-point function of  $F_{SYM}$

$$\begin{aligned} \langle \Omega | T F_{SYM}(x) F_{SYM}(y) | \Omega \rangle \\ \propto \langle \Omega | T (\text{Tr } F_{\mu\nu} F^{\mu\nu} - i F_{\mu\nu} \tilde{F}^{\mu\nu})(x) (\text{Tr } F_{\mu\nu} F^{\mu\nu} - i F_{\mu\nu} \tilde{F}^{\mu\nu})(y) | \Omega \rangle + \dots \end{aligned} \quad (1.2)$$

is fundamentally different from (1.1): (1.2) is the two-point function of dynamical variables, the glue-ball operators. It cannot be ultra-local as (1.1) and in fact supersymmetry does not at all dictate this.

Therefore the relevant question is not, which operators represent the glue-balls in SYM, but how the glue-ball operators in  $F_{SYM}$  can appear in an effective action *dynamically*. Based on an extension of the Veneziano-Yankielowicz Lagrangian first proposed in ref. [15], the present authors showed in refs. [16, 17, 18] how to achieve this. The resulting action has four bosonic degrees of freedom from  $\varphi$  and  $F$ , but only two fermionic ones. The purpose of the present paper is to illuminate this mismatch by the study of the induced supersymmetry transformations of  $S$  in both, the superspace representation as well as the Wess-Zumino gauge. It is found, that a resolution requires a much larger context, including the whole superconformal structure embedded in a supergravity background.

## 2 An Example: The Wess-Zumino Model

To clarify the important difference in the symmetry content of single and composite effective fields resp., the basic ideas are worked out for the Wess-Zumino model in a first step. The absence of a gauge-symmetry drastically simplifies the technical aspects, but at the same time all relevant issues are still reproduced. Nevertheless, this section should be seen as an exercise preparing the more complicated case of SYM. We do not claim that all steps performed here have the same meaning within the Wess-Zumino model.

### 2.1 Simple Effective Fields

We consider the Wess-Zumino model with a single chiral field  $\Phi = \varphi + \theta\psi + \theta^2 F$  and the Lagrangian

$$\mathcal{L}_{Wess-Zumino} = \int d^4\theta \bar{\Phi}\Phi - \left( \int d^2\theta \frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3 + \text{h. c.} \right). \quad (2.1)$$

The effective action may be written in terms of the simple low-energy fields  $\Phi_{eff} = \langle \Omega(J) | \Phi | \Omega(J) \rangle$ , which is obtained from the source extension of the action

$$\mathcal{L}_J = \mathcal{L}_{Wess-Zumino} + \left( \int d^2\theta J\Phi + \text{h. c.} \right) \quad (2.2)$$

and the subsequent Legendre transformation<sup>3</sup>

$$\Gamma[\Phi_{eff}, \bar{\Phi}_{eff}] = \int d^4x \left( \int d^2\theta J\Phi_{eff} + \text{h. c.} \right) - W[J, \bar{J}] . \quad (2.3)$$

By the definition (2.3) the effective action is a integral over superspace as well and thus takes the form<sup>4</sup>

$$\Gamma[\Phi_{eff}, \bar{\Phi}_{eff}] = \int d^4x \left( \int d^4\theta K(\Phi_{eff}, \bar{\Phi}_{eff}) - \left( \int d^2\theta W(\Phi_{eff}) + \text{h. c.} \right) \right) . \quad (2.4)$$

The classical action (2.1) is invariant under the supersymmetry transformations

$$\delta\varphi = \epsilon\psi , \quad \delta\psi_\alpha = \epsilon_\alpha F - i(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \varphi , \quad \delta F = -i\bar{\epsilon}\bar{\sigma}^\mu \partial_\mu \psi \quad (2.5)$$

and they are inherited by the effective fields

$$\delta\varphi_{eff} = \epsilon\psi_{eff} , \quad \delta\psi_{eff\alpha} = \epsilon_\alpha F_{eff} - i(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \varphi_{eff} , \quad \delta F_{eff} = -i\bar{\epsilon}\bar{\sigma}^\mu \partial_\mu \psi_{eff} . \quad (2.6)$$

Supersymmetry covariance of the source extension is manifest as the action (2.2) is invariant under that symmetry if the components of  $J = j + \theta\eta + \theta^2 f$  transform as a chiral field as well.

On the classical level the masses of  $\varphi$  and  $\psi$  are degenerate. It is important to realize that this does not follow from the superpotential alone, but the full potential

$$V = -\bar{F}F + \left( m(F\varphi - \frac{1}{2}\psi\psi) + \lambda(F\varphi^2 - \varphi\psi\psi) + \text{h. c.} \right) - (jF + f\varphi - \eta\psi + \text{h. c.}) \quad (2.7)$$

is important. Only after elimination of the auxiliary field

$$F_{elim} = \bar{m}\bar{\varphi} + \bar{\lambda}\bar{\varphi}^2 - \bar{j} \quad (2.8)$$

the mass of the scalar field becomes manifest. The superpotential alone as a holomorphic function must have unstable directions in the complex fields  $\varphi$  and  $F$ . Therefore the masses of the theory are not defined without detailed knowledge of the Kähler potential.

After the elimination of  $F$  the symmetry transformations (2.5) become (for  $J = 0$ )

$$\delta\varphi = \epsilon\psi , \quad \delta\psi_\alpha = \epsilon_\alpha F_{elim} - i(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \varphi , \quad \delta F_{elim} = \bar{m}\bar{\epsilon}\bar{\psi} + 2\bar{\lambda}\bar{\epsilon}\bar{\psi}\bar{\varphi} . \quad (2.9)$$

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<sup>3</sup>All low energy actions in this work are defined as associated limits of effective actions, valid over the entire range of momenta and externally prescribed field. However, the main conclusions do not rely on this definition, but only on the fact that a supersymmetry covariant set of effective operators may be defined as those, which couple to a source term (local coupling). This applies to any other low energy approximation (as e.g. the Wilsonian action) as well, though no Legendre transformation is performed in these cases.

<sup>4</sup>Of course, holomorphicity arguments in that particular case reduce the effective superpotential to its classical form. As this simplification could hide the relevant observations, we keep a general effective superpotential in the following.

The transformation of the dependent field  $F_{elim}$  is equivalent to  $\delta F$  in (2.5) up to equations of motion.

The purpose of the present paper is, to clarify the rôle of the elimination of auxiliary fields in the context of effective actions. On the one hand the classical theory shows already that a similar procedure can be important for the effective actions as well, namely if  $|W_{,\varphi}|^2$  shall determine the scalar potential. On the other hand, the non-holomorphic part of the effective action need not be restricted to a simple form as in (2.1), in particular there could appear derivative terms on the “effective auxiliary field”, which promotes the latter to a propagating degree of freedom. To avoid misunderstandings in the discussion of different types of “auxiliary” fields we introduce the following nomenclature [19]:

1. The auxiliary fields of the fundamental theory ( $F$  in the present case) are called 1<sup>st</sup> generation or fundamental auxiliary fields.
2. Fields in an effective multiplet that appear at a place where one usually expects an auxiliary field are called 2<sup>nd</sup> generation or effective “auxiliary” fields. The word “auxiliary” is put in quotation marks, as these fields need not have the typical behavior of an auxiliary field, but can contain propagating modes.

In the first place it is important to realize that 2<sup>nd</sup> generation auxiliary fields cannot be eliminated in the sense of (2.8). The number of effective fields does not depend on the number of fundamental fields, which changes by the elimination (2.8), but only on the number of sources. For a chiral field this number is always three, there exists no concept of “elimination of a source”, which would maintain the supersymmetry covariance<sup>5</sup>. The source  $j$  seems to disappear from the potential after elimination of  $F$  as

$$V = F_{elim}\bar{F}_{elim} - \left(\left(\frac{m}{2} + \lambda\varphi\right)\psi\psi + \text{h. c.}\right) - (f\varphi - \eta\psi + \text{h. c.}) . \quad (2.10)$$

But while the action is supersymmetry covariant with all three sources  $j$ ,  $\eta$  and  $f$  this is not the case after reducing the sources to  $\eta$  and  $f$  only, as (2.8) depends on  $j$ .

Nevertheless, in the context of the present example there *must* exist a concept of the elimination of the 2<sup>nd</sup> generation auxiliary field: Indeed, this field (in this particular example) is just the low energy version of the fundamental auxiliary field and thus should inherit its fundamental properties. Indeed, from (2.8) follows after the elimination of the fundamental auxiliary field

$$F_{eff}(x) = \frac{\delta}{\delta j(x)} W[j, \eta, f; \bar{j}, \bar{\eta}, \bar{f}] = \langle \Omega(J) | \bar{m}\bar{\varphi} + \bar{\lambda}\bar{\varphi}^2 | \Omega(J) \rangle [(J)] , \quad (2.11)$$

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<sup>5</sup>Local coupling constants are a powerful tool to investigate supersymmetric theories in  $x$ -space [20, 21, 22, 23]. But as the construction of local couplings is closely related to the formulation in superspace, supersymmetry is enforced in the enveloping superspace, but not reducible to the evaluation of supergraphs.

i.e.  $F_{\text{eff}}$  becomes a function of the fields  $\varphi_{\text{eff}}$  and  $\psi_{\text{eff}}$ . Thus it is necessary and consistent to “eliminate” the effective auxiliary field as well and to postulate

$$\langle \Omega(J) | \bar{m}\bar{\varphi} + \bar{\lambda}\bar{\varphi}^2 - \bar{j} | \Omega(J) \rangle \approx \frac{1}{g_{\varphi\bar{\varphi}}} (\bar{W}_{,\bar{\varphi}} + \frac{1}{2} g_{\varphi\bar{\varphi},\bar{\varphi}} \psi_{\text{eff}} \psi_{\text{eff}}) , \quad (2.12)$$

where  $g_{\varphi\bar{\varphi}}$  is the Kähler metric of the non-holomorphic part in (2.4). This “elimination” has to be understood such that the relation

$$\frac{\delta}{\delta j(x)} W[j, \eta, f; \bar{j}, \bar{\eta}, \bar{f}] = F_{\text{eff}}(x) \quad (2.13)$$

may be replaced by a function of variations with respect to the sources  $\eta, f$  and their hermitian conjugates. It does not mean that  $F_{\text{eff}}$  disappears from the theory, as the variation

$$\frac{\delta}{\delta F_{\text{eff}}(x)} \Gamma[\varphi_{\text{eff}}, \psi_{\text{eff}}, F_{\text{eff}}, \bar{\varphi}_{\text{eff}}, \bar{\psi}_{\text{eff}}, \bar{F}_{\text{eff}}] = j(x) \quad (2.14)$$

must retain its validity.

To conclude, the close relation between the 1<sup>st</sup> and the 2<sup>nd</sup> generation auxiliary fields induces the “elimination” of the latter as a consequence of the elimination of the former using the algebraic equations of motion. This dictates that the equations of motion for  $F_{\text{eff}}$  in (2.4) are algebraic as well, i.e.  $K(\Phi_{\text{eff}}, \bar{\Phi}_{\text{eff}})$  is a Kähler manifold. Still, as outlined above, the elimination of the 1<sup>st</sup> generation auxiliary field and functional restriction of the 2<sup>nd</sup> generation auxiliary field in terms of the physical effective fields are two different procedures. This is relevant to understand the subsequent sections, even if it may appear hair-splitting.

In the generic case the relation (2.12) can be complicated, as  $\langle \Omega | \varphi^2 | \Omega \rangle \neq (\langle \Omega | \varphi | \Omega \rangle)^2$  in general. However, in the present example holomorphicity arguments restrict the effective superpotential to the classical superpotential, which once again justifies the above line of arguments.

## 2.2 Composite Effective Fields

The situation changes drastically, once the source is not coupled to a field monomial, but rather to a composite operator. As an example we consider the action (2.1) together with a source extension coupled to  $\Phi^2 = \Psi$ , only:

$$\mathcal{L}_J = \mathcal{L}_{\text{Wess-Zumino}} + \left( \int d^2\theta J \Phi^2 + \text{h. c.} \right) \quad (2.15)$$

The new effective field  $\Psi_{\text{eff}} = \phi_{\text{eff}} + \theta \kappa_{\text{eff}} + \theta^2 D_{\text{eff}}$  follows from variation with respect to the source  $J$  in (2.15). The field content of its components reads

$$\phi_{\text{eff}} = \langle \Omega(J) | \varphi^2 | \Omega(J) \rangle , \quad (\kappa_{\text{eff}})_\alpha = \langle \Omega(J) | 2\varphi \psi_\alpha | \Omega(J) \rangle , \quad D_{\text{eff}} = \langle \Omega(J) | 2\varphi F - \psi \psi | \Omega(J) \rangle . \quad (2.16)$$

Of course, the three fields transform as a chiral field as may be tested easily by applying the transformation rules (2.5) to the operators in (2.16).

As in the previous section the effect of the elimination of the fundamental auxiliary field on the new effective action<sup>6</sup>

$$\Gamma[\Psi_{eff}, \bar{\Psi}_{eff}] = \int d^4x \left( \int d^4\theta K(\Psi_{eff}, \bar{\Psi}_{eff}) - \left( \int d^2\theta W(\Psi_{eff}) + \text{h. c.} \right) \right) \quad (2.17)$$

must be studied.

From eq. (2.8) the three operators in  $\Psi$  after elimination of  $F$  become ( $J = 0$ ):

$$\phi = \varphi^2 \quad \kappa_\alpha = 2\varphi\psi_\alpha \quad D_{elim} = 2\bar{m}|\varphi|^2 + 2\bar{\lambda}|\varphi|^2\bar{\varphi} - \psi\psi \quad (2.18)$$

The supersymmetry transformations

$$\delta\phi = \epsilon\kappa, \quad \delta\kappa_\alpha = \epsilon_\alpha D_{elim} - i(\bar{\sigma}^\mu\bar{\epsilon})_\alpha\partial_\mu\phi, \quad (2.19)$$

are still functions of the operators<sup>7</sup> in  $\Psi_{elim}$ , while  $D_{elim}$  transforms as

$$\delta D_{elim} = (2\bar{m}\varphi + 4\bar{\lambda}|\varphi|^2)\bar{\epsilon}\bar{\psi} - 2i\bar{\epsilon}\bar{\sigma}^\mu\psi\partial_\mu\varphi. \quad (2.20)$$

Eqs. (2.19) and (2.20) unravel several important differences compared to the case of simple effective fields of the previous section:

If the effective fields are composite operators of the fundamental ones, then the elimination of the fundamental auxiliary field does not induce the elimination of any of the effective fields. Indeed,  $D_{elim/eff} = \langle \Omega(J) | D_{elim} | \Omega(J) \rangle$  cannot be written as a function of  $\phi_{eff}$  and  $\kappa_{eff}$ .

If, accidentally, an ansatz for the effective action  $\Gamma[\Psi_{eff}, \bar{\Psi}_{eff}]$  had algebraic equations of motion for  $D_{elim/eff}$ , this ansatz would fail to capture fundamental properties of the system: Elimination of  $D_{elim/eff}$  would lead to an action formulated solely in terms of  $\phi_{eff}$  and  $\kappa_{eff}$ , but this action cannot be supersymmetric as supersymmetry never closes on these two fields. Moreover, such an action would be ultra-local in  $D_{elim/eff}$ , which is easily falsified:  $D_{elim/eff}$  represents *propagating* degrees of freedom.

This leads to the definite conclusion: *If the effective action of a supersymmetric theory is formulated in terms of composite operators only, then there exist no auxiliary fields among the variables of this action. All fields must be interpreted as (independent) propagating modes.*

Yet the present example seems to be inconsistent: Clearly supersymmetry does not close on the set of operators in  $\Psi_{eff}$  once the fundamental auxiliary field has been eliminated. However, this elimination should have no influence on the physics. Therefore, in all applications of composite effective supermultiplets supersymmetry covariance must be restored due to a complete set of additional relations.

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<sup>6</sup>This effective action is defined for illustrative purposes, only. We do not intend that the full dynamics of the Wess-Zumino model are correctly reproduced by this action.

<sup>7</sup>We denote the set of operators  $(\phi, \kappa, D)$  and the superfield  $\phi + \theta\kappa + \theta^2 D$  by the same symbol  $\Psi$ . The meaning should be clear within the context.

### 3 SUSY representation of confined SYM

The choice of sources for composite operators is important in  $N = 1$  SYM theory, as this theory exhibits confinement. The relevant composite superfield  $S$  (the Lagrangian or anomaly multiplet) is best defined through the anomalous current conservation

$$\bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} = \delta_{\alpha} , \quad \delta_{\alpha} = -2D_{\alpha} S , \quad \bar{D}_{\dot{\alpha}} S = 0 , \quad (3.1)$$

$$S = c \operatorname{Tr} W^{\alpha} W_{\alpha} , \quad c = -\frac{\beta}{24g^3 C(G)} . \quad (3.2)$$

In terms of the physical fields  $A_{\mu}$  (gluon),  $\lambda_{\alpha}$  (gluino) and the real auxiliary field  $D$  the components of  $S = \varphi + \theta\psi + \theta^2 F$  take the form

$$\varphi = 2c \operatorname{Tr} \lambda \lambda \quad (3.3)$$

$$\psi_{\alpha} = \sqrt{2}c (2 \operatorname{Tr} \lambda_{\alpha} D - \operatorname{Tr}(\sigma^{\mu\nu} \lambda)_{\alpha} F_{\mu\nu}) \quad (3.4)$$

$$F = 4ic \operatorname{Tr} \lambda \sigma^{\mu} D_{\mu} \bar{\lambda} - c(\operatorname{Tr} F_{\mu\nu} F^{\mu\nu} - i \operatorname{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}) + 2c \operatorname{Tr} D^2 \quad (3.5)$$

The classical Lagrangian is proportional<sup>8</sup> to  $S$

$$\mathcal{L}_{SYM} = \int d^2\theta \tau_0 \tilde{S} + \text{h. c.} , \quad S = -\frac{\beta}{3g^3} \tilde{S} \quad (3.6)$$

with the complex coupling constant  $\tau_0 = \frac{1}{g^2} + \frac{i\theta}{8\pi^2}$ . An effective action in terms of the field  $S_{eff} = \langle \Omega(J) | S | \Omega(J) \rangle$  is obtained by the extension of  $\tau$  to a chiral superfield  $J = \tau + \theta\eta - 2\theta^2 m$  and a subsequent Legendre transformation with respect to this source [15, 24, 25, 19].

From the supersymmetry transformations<sup>9</sup>

$$\delta A_{\mu} = \frac{1}{\sqrt{2}} (\bar{\epsilon} \bar{\sigma}_{\mu} \lambda + \bar{\lambda} \bar{\sigma}_{\mu} \epsilon) \quad (3.7)$$

$$\delta \lambda_{\alpha} = \frac{1}{\sqrt{2}} \epsilon_{\alpha} D + \frac{1}{2\sqrt{2}} (\sigma^{\mu\nu})_{\alpha} F_{\mu\nu} \quad (3.8)$$

$$\delta D = -\frac{i}{\sqrt{2}} (\epsilon \sigma^{\mu} D_{\mu} \bar{\lambda} + \bar{\epsilon} \sigma^{\mu} \bar{D}_{\mu} \lambda) \quad (3.9)$$

it follows that  $S$  transforms as a chiral superfield as long as  $D$  has not been eliminated. The same should then apply to the effective field  $S_{eff}$  and thus the effective action of SYM takes the form

$$\Gamma_{SYM} = \int d^4\theta K(S_{eff}, \bar{S}_{eff}) - \left( \int d^2\theta W(S_{eff}) + \text{h. c.} \right) . \quad (3.10)$$

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<sup>8</sup>In effective calculations the definition of  $S$  as given in (3.1) should be used, as this quantity is renormalization group invariant. However, in purely classical considerations below the quantity  $\tilde{S}$  is used.

<sup>9</sup>As all quantities used in this work are gauge-invariant, there appear no difficulties in the use of supersymmetry transformations in Wess-Zumino gauge together with the superspace formulation.



The partially anomalous current conservation (3.1) determines the superpotential [14, 15, 24]  $W(S_{\text{eff}}) \propto S_{\text{eff}}(\log S_{\text{eff}}/\Lambda^3 - 1)$ , where  $\Lambda$  is the –in general complex– scale of the theory.

From (3.6) with (3.5) it is easily seen that the auxiliary field is eliminated by the trivial equation of motion  $D = 0$ . After this elimination the new operators

$$\varphi_{\text{elim}} = \varphi , \quad \psi_{\text{elim}} = -\sqrt{2}c \text{Tr}(\sigma^{\mu\nu}\lambda)F_{\mu\nu} , \quad F_{\text{elim}} = F - 2c \text{Tr} D^2 \quad (3.11)$$

transform as

$$\delta\varphi_{\text{elim}} = \epsilon\psi_{\text{elim}} , \quad (3.12)$$

$$\delta(\psi_{\text{elim}})_\alpha = \epsilon_\alpha F_{\text{elim}} - i(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \varphi_{\text{elim}} + 2c \text{Tr} \lambda_\alpha (\epsilon \sigma^\mu D_\mu \bar{\lambda}) + 2c \text{Tr} \lambda_\alpha (\bar{\epsilon} \bar{\sigma}^\mu D_\mu \lambda) , \quad (3.13)$$

$$\delta F_{\text{elim}} = -i\bar{\epsilon}\bar{\sigma}^\mu \partial_\mu \psi_{\text{elim}} . \quad (3.14)$$

As was expected from the analysis of the previous section, supersymmetry does not close on the operators in  $S$  once the auxiliary field has been eliminated. This raises the question about the meaning of the effective action (3.10). This ansatz can be justified in the special case of SYM at least within a restricted range. Indeed it follows from the explicit expression for the supercurrent in (3.1)

$$V_{\alpha\dot{\alpha}} = \frac{1}{C(G)} \text{Tr} W_\alpha e^{-V} \bar{W}_{\dot{\alpha}} e^V \quad (3.15)$$

that the new contributions, which appear on the right hand side of eq. (3.13), are components of  $\bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}$ . Equation (3.1) then suggests the operator relations

$$\lambda_\alpha (D_\mu \lambda \sigma^\mu)_{\dot{\alpha}} = \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \varphi_{\text{elim}} , \quad (3.16)$$

$$\lambda_\alpha (D_\mu \bar{\lambda} \bar{\sigma}^\mu)^\beta = i\delta_\alpha^\beta F_{\text{elim}} . \quad (3.17)$$

Classically, which means  $\delta_\alpha \rightarrow 0$  in (3.1), the right hand side of the eqs. (3.16) and (3.17) is zero and thus the Lagrangian multiplet transforms with the linear transformations of a chiral field even after the auxiliary field  $D$  has been eliminated! At the quantum level the naive application of eqs. (3.16) and (3.17) would lead to the conclusion that

$$\delta(\psi_{\text{elim}})_\alpha = (\epsilon_\alpha F_{\text{elim}} - i(\sigma^\mu \bar{\epsilon})_\alpha \partial_\mu \varphi_{\text{elim}}) \left(1 + \frac{1}{12} \frac{\beta}{g^3}\right) . \quad (3.18)$$

However, one should be more careful with the interpretation of these calculations. In contrast to the Wess-Zumino model a naive elimination of the auxiliary field  $D$  in SYM does not make sense at the quantum level. In Wess-Zumino gauge supersymmetry transformations mix with the gauge symmetries and thus the BRST construction is more involved [26, 27, 28, 29]. Nevertheless, the simple considerations of this section show that –due to the basic structure of the anomalous current conservation– the supersymmetry transformations of the components

of  $S$  close on this set of operators even if the auxiliary field  $D$  has been eliminated. The true transformation rules for the latter case demand a careful quantization in Wess-Zumino gauge, which should follow along the lines of refs. [30,31]. Though it is expected that this leads to a modification of our result, the appearance of quantum deformations of the supersymmetry transformations (as in (3.18)) cannot be excluded. The exact answer and its influence on the anomaly structure (3.1) must be left open in the present work.

We conclude this section with the two main statements, which follow from the considerations made so far:

1. At least for vanishing sources  $J$ , supersymmetry appears to close on the components of the composite superfield  $S$ . This is obvious in the superspace representation, but by means of the anomalous current conservation it holds after the elimination of  $D$  as well.
2. This seems to justify the ansatz for an effective action in terms of the field  $S_{eff}$  alone. Nonetheless the calculation clearly shows that no component of  $S_{eff}$  is an auxiliary field. Thus the effective potential in (3.10) must be bounded from below in *all* fields and at the same time derivatives on  $F_{eff}$  are necessary as well. This is impossible if  $K(S_{eff}, \bar{S}_{eff})$  is a Kähler manifold, but requires a more general ansatz [15, 16, 17, 32].

## 4 Soft Supersymmetry Breaking in SYM

There remains a surprising characteristic of the effective action found in the previous section: This action is formulated in terms of two complex scalar fields  $\varphi$  and  $F$  and one Majorana spinor  $\psi$ . As both scalars must be interpreted as propagating degrees of freedom, on-shell this action has four bosonic and two fermionic degrees of freedom. This seems to contradict supersymmetry, where the number of bosonic and fermionic degrees of freedom should be equal. We are not able to present a conclusive solution to this question, but the study of softly broken supersymmetry shows that it must be seen in a larger context than discussed so far.

Softly broken supersymmetry is obtained in the formulation of the previous section by taking a non-zero, constant limit of the source  $J$ . The complete classical action from (3.6) with  $J$  as defined below that equation takes the form

$$\mathcal{L}_{SYM} = \int d^2\theta (\tau_0 + J)\tilde{S} + \text{h.c.} = (\tau_0 + \tau)\tilde{F} - \eta\tilde{\psi} - 2m\tilde{\varphi} + \text{h.c.} \quad . \quad (4.1)$$

The contribution  $\propto m$  is normalized in such a way that it leads to the standard mass term for the gluino,  $\tau_0$  is the bare coupling constant of eq. (3.6).

For  $J = 0$  the global symmetries are represented by the anomalous current conservation (3.1). In the presence of a non-vanishing source  $J$  it is advantageous to redefine the current

$V_{\alpha\dot{\alpha}}$  of eq. (3.15) as

$$V_{\alpha\dot{\alpha}} = \frac{1}{2C(G)}(\tau_0 + \bar{\tau}_0) \text{Tr } W_{\alpha} e^{-V} \bar{W}_{\dot{\alpha}} e^V . \quad (4.2)$$

Then the breaking terms take the form [33]

$$\bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} = -2(D_{\alpha} J) \tilde{S} + \frac{1}{\tau_0 + \bar{\tau}_0} (\bar{D}^{\dot{\alpha}} \bar{J}) V_{\alpha\dot{\alpha}} + \frac{J + \bar{J}}{\tau_0 + \bar{\tau}_0} \bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} + \delta_{\alpha} . \quad (4.3)$$

The physically most interesting situation is certainly soft supersymmetry breaking due to a finite mass of the gluino, which means  $J = -2\theta^2 m$ , whereby  $m$  is constant. As  $m$  does not couple to  $\tilde{\psi}$  in (4.1) the equation of motion for  $D$  still reads  $D = 0$ . Thus the components of  $S_{elim}$  still transform according to (3.12)-(3.14). But due to the right hand side of eq. (4.3), the modification of the transformation law of  $\tilde{\psi}$  in (3.13) do no longer vanish even setting  $\delta_{\alpha} = 0$ . Instead the eqs. (3.16) and (3.17) are changed according to

$$\lambda_{\alpha} (D_{\mu} \lambda \sigma^{\mu})_{\dot{\alpha}} = -i \frac{2m}{\tau_0 + \bar{\tau}_0} R_{\alpha\dot{\alpha}} + \text{quantum corrections} , \quad (4.4)$$

$$\lambda_{\alpha} (D_{\mu} \bar{\lambda} \bar{\sigma}^{\mu})^{\beta} = -\frac{i}{2} \delta_{\alpha}^{\beta} m \tilde{\varphi}_{elim} + \text{quantum corrections} , \quad (4.5)$$

with the  $R$  current  $R_{\alpha\dot{\alpha}}$  defined as the lowest component of (4.2). This leads to the transformation rule

$$\delta(\tilde{\psi}_{elim})_{\alpha} = \epsilon_{\alpha} (\tilde{F}_{elim} + \frac{1}{2} m \tilde{\varphi}_{elim}) - i(\sigma^{\mu} \bar{\epsilon})_{\alpha} \partial_{\mu} \tilde{\varphi}_{elim} + \frac{m}{4(\tau_0 + \bar{\tau}_0)} R_{\alpha\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}} + \text{quantum corrections} . \quad (4.6)$$

In eq. (4.6) the deformation of the transformation rule from superspace is not just a result of the quantization, but contains classical contributions as well.

Of course any deformation of the superspace transformation rules immediately raises the question how to construct an invariant action respecting the “on-shell” symmetry transformations. This certainly applies to the result of eq. (4.6) as well, but, even more important, this equation shows that an effective action in terms of  $S_{eff}$  alone is not complete. There exists no operator identity that would transform the  $R$  current in eq. (4.6) into an expression in terms of the components of  $S_{elim}$  and consequently supersymmetry does not close on the operators of  $S_{elim}$ , at least if the gluino mass does not vanish<sup>10</sup>.

This result demonstrates that an equal number of fermionic and bosonic degrees of freedom can be found in a larger context, only. To include besides the anomaly multiplet  $S$  the

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<sup>10</sup>One might wonder why we still insist on correct supersymmetry transformation rules, once this symmetry is broken anyway. However, it is important to realize that the action (4.1) is invariant under supersymmetry, if the sources  $J$  are transformed as well. This conserved supersymmetry has its own current and the result (4.6) could have been derived therefrom as well. The shortcut of using the partially conserved current is allowed, as  $\tilde{\psi}_{elim}$  does not depend on the sources and moreover we think that the breaking terms are more transparent in this way.

supercurrent  $V_{\alpha\dot{\alpha}}$  as field relevant for the effective action, was proposed in [15] already. But the ansatz in ref. [15] led to complicated differential equations in superspace, which could not be solved. Within the perturbative framework, some relevant results have been derived in [22, 23, 34]. It is not the purpose of this paper to investigate the technical details of such a formulation, but we conclude this section with some comments on the difficulties in the construction of a consistent effective action:

- $S$  and  $V_{\alpha\dot{\alpha}}$  do not restore an equal number of bosonic and fermionic degrees of freedom.
- For non-vanishing gluino mass supersymmetry closes on the operators in  $S$  and  $V_{\alpha\dot{\alpha}}$ , but this is not true for non-vanishing fermionic source  $\eta_\alpha$ . We were not able to find a definite interpretation of the rather complicated formulas, which follow in that case.
- Both points indicate that an even more general field and source content is necessary:  $J^{\alpha\dot{\alpha}}$  needed to couple the supercurrent  $V_{\alpha\dot{\alpha}}$  to the SYM action contains a source for the energy-momentum tensor. Thus the system must be embedded in a non-trivial supergravity background together with all currents of the super-conformal structure. Only within this maximal set of gauge invariant and supersymmetry covariant fields a resolution of the supersymmetry representation on composite effective fields can be expected. On the way there, the fate of the different constraints on the superfields, which are crucial for the formulation of the classical theory, must be considered at the quantum level. This amounts to study an anomaly structure within a more general context than done in (3.1) including eventual non-vanishing  $\delta_\alpha$ .

## 5 Conclusions

The elimination of auxiliary fields is an important procedure in all supersymmetric theories that allow a superspace formulation. At the level of the classical Lagrangian its meaning is obvious: Elimination of an auxiliary fields means to impose a constraint, which holds the field at the extremum (maximum) of its potential. The physics of the theory do not change under this elimination.

At the level of effective actions the situation is more involved: There exist no separate constraints for the effective fields, but an eventual elimination must follow from the constraints on the fundamental fields. The result of these considerations depends on the details of the effective fields:

- If the effective fields are simple operators, i.e. the sources couple to field monomials, the elimination of the fundamental auxiliary fields induces the functional restriction of the effective auxiliary fields. The effective action as well as the supersymmetry transformations may be written in terms of the remaining fields.

- If the effective fields are composite operators, the elimination of the fundamental auxiliary field leads to a change in the field content of the effective fields as well. But none of the resulting effective fields may be written as a function of the other ones. Thus there exists no concept of the elimination of fields in an effective action solely written in terms of composite effective fields.

The supersymmetry transformations lead to an even more stringent condition: The “on-shell” supersymmetry transformations do not close on a subset of the effective fields, which confirms the above statement. But in the generic case they do not even close on the composite fields stemming from those superfields, which were used to define the effective action. If this happens the formulation in superspace must be inconsistent as well, as it must be physically equivalent to the non-linear “on-shell” formulation.

In the second part of the paper we analyzed the effective action of  $N = 1$  SYM from this point of view:

- At the classical level ( $\delta_\alpha = 0$  in eq. (3.1)) the formulation of the effective action in terms of  $S$  alone appears to be consistent within a restricted range:
  1. For vanishing sources supersymmetry closes even after the elimination of  $D$  on the three operators in  $S$ . The supersymmetry transformations do not experience any change by this elimination. Clearly, no field in  $S$  can be treated as an auxiliary field.
  2. Softly broken supersymmetry, which is equivalent to a non-zero but constant source, cannot be described in terms of  $S$  alone. E.g. a soft gluino mass induces supersymmetry transformations into the  $R$  current. Thus at least the supercurrent must be coupled with a source as well. This dictates to consider the model in a supergravity background, as it contains a source for the energy-momentum tensor.
- At the quantum level a deformation of the classical “on-shell” supersymmetry transformations must be expected, even without soft supersymmetry breaking.

These important characteristics are certainly a serious challenge for our present knowledge about  $N = 1$  SYM theory. To study its consequences necessitates investigations in many directions, esp. the results will never be found within the purely holomorphic sector of the effective action.

## Acknowledgement

The authors would like to thank E. Kraus and Ch. Rupp for important discussions. One of us (L.B.) would like to thank J.-P. Derendinger, U. Ellwanger and E. Scheidegger for interesting comments. Important parts of this work have been developed on the occasion of a workshop at the International Erwin Schrödinger Institute. This work has been supported by the Swiss

National Science Foundation (SNF) and the Austrian Science Foundation (FWF) project P-16030-N08.

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